Infinitesimal and tangential 16-th Hilbert problem on zero-cycles

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Classical infinitesimal and tangential 16-th Hilbert problem deal with deformations of integrable vector fields in the plane; $dF + \epsilon\omega = 0$, with $F \in \mathbb{C}[x, y]$ and ω a polynomial 1-form. The infinitesimal problem asks for the number of limit cycles born in the deformation, while the tangential problem is the question about the number of zeros of Abelian integrals; that is, the isolated zeros of the displacement function $\Delta(t, \epsilon) = \epsilon M_1(t) + o(\epsilon)$ and isolated zeros of $M_1(t) = -\int_{\gamma(t)\subset F^{-1}(t)} \omega$, respectively. These two numbers can differ by the presence of the so-called Alien cycles.

In this talk we will consider the analogous questions in zero-dimension. More precisely, given a polynomial f in one complex variable, we consider a zero-cycle $C(t) = \sum_{j=1}^{m} n_j z_j(t)$ defined by the roots $z_j(t)$ of $f = t$. Then we perform a deformation $f + \epsilon g$, with g a polynomial and ϵ a small parameter, and consider the deformed zero-cycle $C_\epsilon(t) = \sum_{j=1}^{m} n_j z_j(t, \epsilon)$. Analogously to the classical problems, the displacement function is defined by $\Delta(t, \epsilon) = \int_{C_\epsilon(t)} f = \sum_{j=1}^{m} n_j f(z_j(t, \epsilon))$, which is equal to $\epsilon M_1(t) + o(\epsilon)$, with $M_1(t) = -\int_{C(t)} g$. Infinitesimal and tangential 16-th Hilbert problem for zero-cycles are problems of counting isolated regular zeros of $\Delta(t, \epsilon)$, for small ϵ, and of $M_1(t)$, respectively. I will present some recent results, of a work in progress, about bounding zeros of Δ and M_1, and the possibility of having Alien cycles.

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